Introductory and advanced courses

Course 1
Title: The mapping class group action on surfaces from the point of view of length spectra
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Viveka Erlandsson, University in Bristol, Bristol, UK
Abstract of the course:

1. Introduction to hyperbolic geometry and constructions of hyperbolic structures on surfaces
2. Counting simple closed geodesics on surfaces: Mirzakhani's work
3. Counting simple closed geodesics on hyperbolic surfaces with singular points.
4. Introduction to spaces of geodesic currents on surfaces: invariant measures and synamics.
5. Counting simple closed geodesics on surfaces. Recent results and open problems.

Course 2
Title: Funk and Hilbert geometry in Euclidean and non-Euclidean geometry
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Sumio Yamada, Gakushuin University, Tokyo, Japan
Abstract of the course:

1. Introduction to Funk and Hilbert geometries.
2. Funk and Hilbert geometries in the non-Euclidean setting.
3. Timelike Funk and Hilbert geometries. Group actions and dynamics.
4. Timelike Funk and Hilbert geometries: The case of the simplex and other Example.
5. Funk geometries and Teichmüller theory: the Weil-Petersson-Funk metric.

Selected bibliography:

1. A. Papadopoulos, and M. Troyanov (ed.), Handbook of Hilbert geometry. IRMA Lectures in Mathematics and Theoretical Physics 22. Zürich: European Mathematical Society (EMS) (2014).

2. A. Papadopoulos and S. Yamada, Timelike Hilbert geometry of the spherical simplex. Papadopoulos, Athanase (ed.), Essays in geometry. Dedicated to Norbert A'Campo. Berlin: European Mathematical Society. IRMA Lect. Math. Theor. Phys. 34, 847-870 (2023).

3. A. Papadopoulos and S. Yamada, Timelike Hilbert and Funk geometries. Differ. Geom. Appl. 67, Article ID 101554, 42 p. (2019).

4. A. Papadopoulos and S. Yamada, Timelike Hilbert geometry of the spherical simplex. In: Papadopoulos, Athanase (ed.), Essays in geometry. Dedicated to Norbert A'Campo. Berlin: European Mathematical Society. IRMA Lect. Math. Theor. Phys. 34, 847-870 (2023).

5. S. Yamada, On the Weil-Petersson convex geometry of Teichmüller space. Sugaku Expo. 30, No. 2, 159-186 (2017).

6. S. Yamada, Convex bodies in Euclidean and Weil-Petersson geometries. Proc. Am. Math. Soc. 142, No. 2, 603-616 (2014).

Course 3
Title: The metric theory of Teichmüller spaces of surfaces
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Athanase Papadopoulos, CNRS and University of Strasbourg, France
Abstract of the course:

1. Quasiconformal mappings, best quasiconformal mappings between Riemann surfaces, metric dilatation of best quasiconformal mappings between hyperbolic and Euclidean surfaces

2. The Thurston metric on the Teichmüller space of hyperbolic surfaces and of Euclidean surfaces.

3. The Finsler structure of the Thurston metric

4. Dynamics and rigidity results for the Thurston metric

5. Thurston's metric: The Euclidean case. The Teichmüller spaces of Euclidean triangles and of quadrangulated surfaces.

Selected bibliography:

1. I. Saglam and A. Papadopoulos, Thurston's asymmetric metric on the space of singular flat metrics with a fixed quadrangulation. Enseign. Math. (2) 70, No. 1-2, 231-250 (2024).

2. Y. Huang, K. Ohshika and A. Papadopoulos, The infinitesimal and global Thurston geometry of Teichmüller space. arXiv:2111.13381 (2021), To appear in Journal of Differential Geometry.

3. A. Papadopoulos and W. Su, On the Finsler structure of Teichmüller's metric and Thurston's metric. Expo. Math. 33, No. 1, 30-47 (2015).

4. A. Papadopoulos, Ideal triangles, hyperbolic surfaces and the Thurston metric on Teichmüller space. In: L. Ji, and S.-T. Yau, Moduli spaces and locally symmetric spaces. Based on two workshops, Morningside Center of Mathematics, Beijing, China, February 2017 and March 2019. Somerville, MA: International Press; Beijing: Higher Education Press. Surv. Mod. Math. 16, 135-181 (2021).

5. I. Saglam and A. Papadopoulos, Minimal stretch maps between Euclidean triangle, Preprint, arXiv:2107.00937 (2021), to appear in Osaka J. of Math.

Course 4
Title: Spaces of geodesic laminations on surfaces and 3-manifolds, actions of mapping class groups.
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Ken'ichi Ohshika, Gakushuin University, Tokyo, Japan
Abstract of the course:

1. The space of measured laminations and the space of unmeasured laminations on a hyperbolic surface. Measure topology. Rigidity of the mapping class group action on these spaces.

2. The space of chain-recurrent laminations. How this space appears in the theiry of the Thurston metric on Teichmüller space.

3. Nonsymmetric Hausdorff topology on the space of chain-recurrent laminations, rational depth of a lamination. Rigidity results for the action of the mapping class group.

4. The space of ending laminations. The ending lamination theorem.

5. Discussion of open problems related to spaces of laminations in hyperbolic geometry.

Selected bibliography:

1. W. P. Thurston, The geometry and topology of 3-manifolds, Lecture Notes, Princeton University, 1986.

2. K. Ohshika and A. Papadopoulos, Bijections of geodesic lamination space preserving left Hausdorff convergence, Monatsh. Math. 189, No. 3, 507-521 (2019).

3. K. Ohshika and A. Papadopoulos, Homeomorphisms and intersection numbers, C. R., Math., Acad. Sci. Paris 356, No. 8, 899-902 (2018).

4. S. Baba and K. Ohshika, Realisation of bending measured laminations by Kleinian surface groups. Int. Math. Res. Not. 2023, No. 19, 16674-16707 (2023).

5. K. Ohshika, Teichmüller spaces and the rigidity of mapping class group actions. In: Papadopoulos, Athanase (ed.), Surveys in geometry I. Cham: Springer. p. 389-415 (2022).

6. K. Ohshika, Geometry of Kleinian groups and its applications. Sugaku Expo. 34, No. 2, 123-139 (2021).

Course 5
Title: Group actions and surfaces
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Pallavi Dani, Louisiana State University, LA, USA
Abstract of the course:

1. Combinatorial group theory in surface theory, the braid group and the mapping class group.

2. Coxeter and Artin groups.

3. Right-angled Artin subgroups of right-angled Coxeter and Artin groups.

4. Stallings techniques for subgroups of right-angled Coxeter groups.

5. Closed curves on surfaces and filling loops at infinity in the mapping class group.

Selected bibliography:

1. P. Dani, The large-scale geometry of right-angled Coxeter groups. In: L. Ji, A. Papadopoulos and S.T. Yau (ed.) Handbook of group actions V. Somerville, MA: International Press; Beijing: Higher Education Press. Adv. Lect. Math. (ALM) 48, 107-141 (2020).

2. P. Dani and I. Levcovitz, Right-angled Artin subgroups of right-angled Coxeter and Artin groups. Algebr. Geom. Topol. 24, No. 2, 755-802 (2024).

3. P. Dani, M. Haulmark, F. Walsh, Right-angled Coxeter groups with non-planar boundary. Groups Geom. Dyn. 17, No. 1, 127-155 (2023).

Course 6
Title: Moduli spaces of linkages
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Alena Zhukova, Saint Petersburg State University, Russia.
Abstract of the course:

1. Introduction to mechanical linkages, Examples from Chebychev to Thurston
2. The universality theorem and Thurston's signature theorem. The works of Kempfe and
3. The Morse theory approach : Signed area of polygons and critical points of the signed area of polygonal linkages.
4. Discrete Morse theory for moduli spaces of flexible polygons
5. The discrete Morse theory for the moduli spaces of general polygonal linkages. Open problems.

Selected bibliography:

1. A. Sossinsky, Configuration spaces of planar linkages. In: Papadopoulos, Athanase (ed.), Handbook of Teichmüller theory. Volume VI. Zürich: European Mathematical Society (EMS). IRMA Lectures in Mathematics and Theoretical Physics 27, 335-373 (2016).
2. M. Farber, Invitation to topological robotics. Zurich Lectures in Advanced Mathematics. Zürich: European Mathematical Society, EMS (2008).
3. G. Panina and A. Zhukova, Discrete Morse theory for the moduli spaces of polygonal linkages, or solitaire on a circle. Sb. Math. 208, No. 9, 1353-1367 (2017).
4. A. Zhukova, Discrete Morse theory for the barycentric subdivision, J. Math. Sci., New York 232, No. 2, 129-137 (2018).

Course 7
Title: Differential geometry and combinatorics of curves on surfaces
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Ingrid Irmer, Southern University of Science and Technology, Guangdong, China
Abstract of the course:

1. The problem of finding simple curves on hyperbolic surfaces. Undecidability of this problem.

2. Systoles on surfaces: The work of Paul Schmutz on filling systoles, and recent results.

3. The Thurston spine for Teichmüller space: Introduction and basic properties.

4. The Differential Topology of the Thurston spine:

5. The Thurston spine: Morse-Smale property.

Selected bibliography:

1. I. Irmer, The Morse-Smale property of the Thurston spine. arXiv:2401.05734, 2024.

2. I. Irmer, The Differential Topology of the Thurston Spine of Teichmüller Space. arXiv:2211.03429, 2022.

3. I. Irmer, Finding simple curves in surface covers is undecidable. Geom. Dedicata 206, 201-210 (2020).

4. I. Irmer, Lifts of simple curves in finite regular coverings of closed surfaces. Geom. Dedicata 200, 67-76 (2019).

5. P. Schmutz, Geometry of Riemann surfaces based on closed geodesics. Bull. Am. Math. Soc., New Ser. 35, No. 3, 193-214 (1998).

6. P. Schmutz, Arithmetic groups and the length spectrum of Riemann surfaces. Duke Math. J. 84, No. 1, 199-215 (1996).

7. W. P. Thurston, A spine for Teichmüller space, preprint, 1986.

Course 8
Title: Finsler geometry from the distance point of view.
Duration: 5 hours
Level: 3 lectures basics and 2 lectures on advanced course.
Speakers's name and affiliation: Professor Marc Troyanov, Swiss Federal Institute of Technology Lausanne, Switzerland.
Abstract of the course:

1. An introduction to metric curvature on surfaces and to the theory of Alexandrov surfaces

2. Alexandrov surfaces of bounded Integral curvature

3. Isoperimetric inequalities

4. The Busemann-Mayers-Kobayashi theorem for Finsler manifolds.

5. Introduction to the Binet Legendre metric.

Selected bibliography:

1. A. Papadopoulos, and M. Troyanov (ed.), Handbook of Hilbert geometry. IRMA Lectures in Mathematics and Theoretical Physics 22. Zürich: European Mathematical Society (EMS) (2014).

2. M. Troyanov, Double Forms, Curvature Integrals and the Gauss–Bonnet Formula, In: Surveys in Geometry II, ed. A. Papadopoulos, Springer, 2024, p. 93-143.

3. V. Matveev and M. Troyanov, Completeness and incompleteness of the Binet-Legendre metric. Eur. J. Math. 1, No. 3, 483-502 (2015).

4. V. Matveev and M. Troyanov, Isometries of two dimensional Hilbert geometries. Enseign. Math. (2) 61, No. 3-4, 453-460 (2015).

5. V. Matveev and M. Troyanov, The Binet-Legendre metric in Finsler geometry. Geom. Topol. 16, No. 4, 2135-2170 (2012).

6. B. L. Santos Correia and M. Troyanov, Isoperimetry in Finitely Generated Groups, In: Surveys in Geometry II, ed. A. Papadopoulos, Springer, 2024, p. 361-386.

4 Complementary activities

Interactive session

Description : The last session of last two days of the workshop will consist of interactive sessions with the participants. Those who will have most actively participated to the program will be asked to give short talks on topics with their choice, the professors and the other students will comment.
Duration : 3 hours

Training sessions

Each teacher for each course has agreed to give one training session, consisting of exercices for the students. Thus, 8 training sessions, of duration 1 and half hour each, will be distributed along the period of the school. Thus the training teachers will be the lecturers of the school, see above. The coordinator of the training sessions, who will help also the students in solving the exercices, will be Dr. Soumya Dey, Assistant Professor, Krea University.