



# National Centre for Mathematics

[www.ncmath.org](http://www.ncmath.org)

(A joint centre of TIFR and IIT Bombay)

## Advanced Training in Mathematics Schools

[www.atmschools.org](http://www.atmschools.org)

(Supported by National Board for Higher Mathematics)



## Online NCM Workshop on Modular Forms (2020)

(December 14 - 19, 2020)



Anupam Saikia,  
Professor  
Dept of Mathematics  
IIT Guwahati, PIN 781039  
email : a.saikia@iitg.ac.in

### Conveners

Rupam Barman  
Associate Professor  
Dept of Mathematics  
IIT Guwahati, PIN 781039  
email : rupam@iitg.ac.in



The aim of the proposed online workshop on Modular Forms to be held at IIT Guwahati is to go beyond the basic theory and provide exposure to somewhat more advanced themes. There was an AIS workshop held at IIT Guwahati in the summer of 2019 that covered mostly the basic aspects of the theory of modular forms. The proposed workshop can be considered as a natural follow-up. One of the themes of the workshop is going to be modular forms of half-integral weight and Shimura correspondence. The work-shop will devote a series of lectures to Hilbert modular forms which constitute a natural generalization of the classical theory. There will also be lectures on the zeroes of modular forms and the extremal values of the modular L-functions. The workshop will introduce the participants to the theory of Lambda-adic forms which has turned out to be a very significant development. In addition, there will be two lectures on Lehmer's conjecture and Gauss's class number problem that will expose the participants to some of the very recent progress.

### Module-1: Half integral weight modular forms

Speaker: Professor B. Ramakrishnan,  
Central Univ. of Tamilnadu (on lien from HRI, Prayagraj)



Abstract: In these lectures we discuss some topics in the theory of modular forms of half-integral weight. After a brief introduction, we discuss the following topics: Hecke operators; Kohnen plus space, Shimura correspondence; and some applications.

### Module-2: Lambda-adic forms

Speaker: Professor Eknath Ghate  
TIFR Mumbai



Abstract: Modular forms are holomorphic functions on the upper-half complex plane with many symmetries. These analytic objects have been studied classically for their own sake by several mathematicians and especially by Shimura, and have many applications to number theory problems. On the other hand, Iwasawa studied the variation of the  $p$ -part of the class groups of cyclotomic fields algebraically

for  $p$  a prime. Hida had the deep insight that these two disparate themes in number theory can be combined. He defined and studied  $p$ -adic families of modular forms, more formally known as Lambda-adic forms. These lectures will be an introduction to these forms, leading up to the construction of their Galois representations.

### Module-3: Zeros of modular forms, Atkin-Serre conjecture, and Extreme values of modular L-functions.

Speaker: Professor Sanoli Gun, IMSc Chennai



Abstract:

First set of lectures will center around the resonance method introduced by Soundararajan and its applications to study extreme values of L-functions of cusp forms. Next we plan to discuss the nature as well as locations of zeros of modular forms and quasi modular forms. Finally, we plan to introduce the Atkin-Serre conjecture and illustrate the application of transcendence theory in this set up.

## Module-4: Hilbert Modular Forms

**Speaker: Dr. Aribam Chandrakant Sharma, IISER Mohali**

Abstract:

We will introduce the space of Hilbert Modular forms over totally real fields. We will then go over to study the ring of Hecke operators on them.



## Module-5:

**Under this module Professor Ken Ono of Univ. of Virginia will deliver two lectures**



### Lecture 1: Lehmer's Conjecture and its Generalisations

Abstract: This talk will be an overview of recent works on Lehmer's Conjecture on the nonvanishing of Ramanujan's tau-function. There has been much work recently on Lehmer's Conjecture and its generalization to arbitrary modular newforms. A new approach has been crafted by combining the Chaubauty-Coleman method, Diophantine approximation, Galois representations, and the "modular method" (i.e. in the spirit of the proof of Fermat's Last Theorem). For the first time one can identify integers which never arise as Fourier coefficients of modular newforms such as Ramanujan's tau-function.

(This lecture will be on joint work with Jennifer Balakrishnan, Will Craig, and Wei-Lun Tsai)

### Lecture 2: Gauss' Class Number Problem

Abstract: This talk will be an overview of Gauss' class number problem for imaginary quadratic fields. For every positive integer  $h$ , Gauss Conjectured that there are at most finitely many fundamental discriminants  $-D$  for which  $h(-D)=h$ . The class number 1 case (i.e. where  $h=1$ ) is the celebrated work of A. Baker, K. Heegner, and H. Stark. For general  $h$ , this problem was not effectively solved until the work of D. Goldfeld, B. Gross, and D. Zagier in the early 1980s. However, their results fall far short of the expected

asymptotic properties. In recent work with M. Griffin, the speaker has obtained stronger lower bounds by introducing a method involving "elliptic curve ideal class pairings". The method makes use of these new pairings, effective results on the number of rational points on elliptic curves with bounded height, and some Diophantine geometry. As a bonus, these ideas lead to new effective lower bounds for the heights of non-torsion points on all rational elliptic curves in the direction Of Lang's Height Conjecture.

**Email address to which communications concerning the application can be sent :**  
**rupam@iitg.ac.in**

**Postal address to which communications concerning the application can be sent :**  
**Prof. Anupam Saikia, Department of Mathematics, IIT Guwahati - 781039, Assam**

Application form and other information about the programme are available on the following website :  
**[www.atmschools.org/2020](http://www.atmschools.org/2020)**

**Last date for receiving completed application is :**  
**November 15, 2020**

**List of selected candidates will be posted on the above mentioned website on :**  
**November 22, 2020**