

National Centre for Mathematics

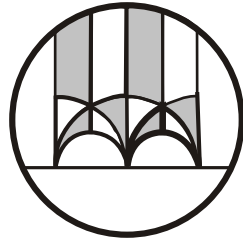
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(A joint centre of TIFR and IIT Bombay)

Advanced Training in Mathematics Schools

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Online Workshop on Finite groups of Lie type

Host Institute : I I S E R Pune

(A follow up of Algebraic Group Workshop held in 2019 at IIT Bombay, Mumbai)

July 31 to October 3, 2021

Organisers: Dr. Anupam Kumar Singh, IISER Pune & Dr. Shripad M Garge, IIT Bombay, Mumbai

Details of the courses:

Course 1 --- Finite Groups of Lie type: This topic would be a natural follow up of the 2019 AIS on Algebraic groups where the theory over algebraically closed fields was considered. This topic intends to expose the participants to the structure of reductive groups defined over a finite field and their representations with emphasis on Deligne-Lusztig theory.

Course 2 --- Word maps and linear groups: It was a conjecture of Oystein Ore, from 1951, that every element of a non-abelian finite simple group G is a commutator. After almost 60 years, this conjecture was settled in affirmative in the celebrated paper of Liebeck, O'Brien, Shalev, and Tiep, well-known in the community as LOST. Ore's conjecture asserts that the function, not a group

homomorphism, from $G \times G$ to G which sends a pair (g, h) to the element $ghg^{-1}h^{-1}$ is surjective. It was then natural to consider various such maps, called word maps, and investigate their properties. The theory of word maps on groups and their applications has developed significantly in the past two decades, with emphasis on finite simple groups, infinite linear groups, profinite groups and related objects. Word width was studied, leading to solutions to a question of Serre (by Nikolov and Segal) and the conjecture of Ore, stated above. Probabilistic aspects of word maps were also explored, leading to a probabilistic Tits alternative for linear groups, characterizations of probabilistically nilpotent groups, as well as recent solutions to probabilistic Waring problems. This series of lectures will describe these developments, related results, and tools used in the proofs.

Speakers & Abstracts

1. Finite Groups of Lie type

Dipendra Prasad, IIT Bombay, India

(Representations of finite groups of Lie type)

This lecture kick-starts the workshop with a brief overview of the celebrated Deligne-Lusztig theory of representations of finite groups of Lie type.

Shripad Garge, IIT Bombay, Mumbai, India

(Structure of finite groups of Lie type)

We aim to prepare the audience for the following two lecture courses. We give a brief review of representations of finite groups of Lie type in the first lecture. In the remaining lectures, we will study the structure of centralisers of elements in finite groups of Lie type.

Manish Mishra, IISER Pune, India

(Supercuspidal representations for finite and p -adic groups)

Let G be a connected reductive group defined over a finite or a non-archimedean local field F . We show that $G(F)$ admits cuspidal representations when F is finite and supercuspidal representations when F is non-archimedean local. We also determine precisely when $G(F)$ admits self-dual representations. For the results on self-duality, we assume some mild hypothesis on G . These hypothesis disallow G to have certain small rank factors when the field (in case F is finite) or the residue field (in case F is non-archimedean local) is of cardinality ≤ 5 .

C. R. Vinroot, College of William and Mary, VA, USA

(Real representations of finite groups of Lie type)

The main purpose of these lectures is to give both an overview and some details on the current state of results on the classification of real representations of finite groups of Lie type

2. Word maps on linear groups

Amit Kulshrestha, IISER Mohali, India

(Images of word maps and chirality)

Let G be a finite simple group. In 2014, Lubotzky proved that if G is a finite simple group then every subset of G which contains identity and is invariant under automorphisms of G , occurs as image set for a suitable word. After a brief idea of the proof, we shall discuss word images in symmetric groups (work of Matthew Levy) and chirality of words (work of Cocke-Ho).

Boris Kunyavskii, Bar-Ilan University, Israel

(Word maps on linear algebraic groups)

Given a group word $w = w(x_1, \dots, x_d)$ on d letters and a group G , one can consider the evaluation map $w: G^d \rightarrow G$ induced by w . In this minicourse, we shall consider the case where $G = G(K)$ is the group of K -points of a linear algebraic K -group G . Our focus is on the special ground fields (complex, real, p -adic, number, or close to such) and on simple K -groups (or close to such). The course is built as a survey of results and open problems concerning the image of a word map, which corresponds to the solvability of the equation $w = w(x_1, \dots, x_d) = g$ for an arbitrary (or 'generic') g , as well as on the fibres of this map, which corresponds to distribution of solutions of such an equation. Various analogies with polynomial maps on associative and Lie algebras will also be discussed.

Anupam Singh, IISER Pune, India

(Revision of algebraic groups)

This lecture will briefly recall the prerequisite for this series of lectures, namely, algebraic geometry, algebraic groups, word maps etc.

(Power maps on finite groups of Lie type)

One of the simplest example of word map is the power map. Let $M \geq 2$ be an integer. Let G be a reductive group defined over a finite field \mathbb{F}_q . We consider the word map $x \mapsto x^M$ on $G(\mathbb{F}_q)$. We study the following question. What is the size of the image under this map? The reference to these lectures will be the work done by the speaker in collaboration with Amit Kulshrestha and Rijubrata Kundu.

B. Sury, ISI Bengaluru, India

(Combinatorial theory of arithmetic groups)

In the 1980's, some abstract and combinatorial group theoretic notions started to be studied in relation to lattices in Lie groups and arithmetic groups in particular. One of them - called Bounded generation - turned out to have unexpectedly strong implications to aspects like representation theory, super-rigidity and the congruence subgroup problem. In these two talks, we survey the developments right up to some very recent ones that address S -arithmetic groups in anisotropic algebraic groups over number fields. The various combinatorial facets of arithmetic groups use interestingly diverse methods.

Tutors: Anirban Bose - IISER Mohali, Sushil Bhunia - IISER Mohali, Pratyusha Chattopadhyay - BITS Pilani, Hyderabad, Shiv Prakash Patel - IIT Delhi

Email address to which soft copies of the applications can be sent: algebraicgroups19@gmail.com

Last date for submission of the online registrations by the applicants : 24th July 2021

The web page of this program will be : <https://www.atmschools.org/school/2021/NCMW/fglt>
Application form and other information about the programme are available on the above website.

The organizers may be contacted by mail: 1anupamk18@gmail.com, smgarge@gmail.com